# Estimation of Effective Atomic Numbers of Polyethylene and Coal Using Compton Scattering

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***Abstract-***In this study, effective atomic numbers of composite materials, polyethylene and coal have been estimated by using Compton scattering of 662keV gamma rays emitted from $^{137}$Cs source. In order to estimate the effective atomic numbers, the Klein-Nishina theoretical cross sections were obtained from $50^0$ to $100^0$ angles. Then, the scattering cross sections of the samples were experimentally determined. By using this method, effective atomic numbers of polyethylene and coal are obtained as 2.25 and 2.39 respectively.

***Keywords-***Compton, Scattering, Effective Atomic Number, Polyethylene, Coal

## I. INTRODUCTION

According to recent researches, the effective atomic number is a measure of the average number of electrons of the material that participate actively during the interaction. It is important in concrete, polymers, and the rare earth compounds [i-vi].

Polyethylene (PE), which is obtained from the polymerization of ethylene, is one of the simplest and most inexpensive polymers and most widely-used plastic material in the world [vii]. Ethylene molecule has a C = C double bond. In the polymerization process, the monomer double bonds are broken, and instead, a simple bond is formed between carbon atoms and N molecules. Coal is composed primarily of carbon hydrogen, sulfur, oxygen, and nitrogen. The typical carbon content for coal (dry basis) varies from 60 percent for lignite to more than 80 percent for anthracite [viii]. Therefore, polyethylene and coal are not pure elements with a reasonable standard for their atomic numbers. One of the various methods used to determine atomic number of composite materials is Compton scattering.

In early 1920, the Compton Effect was observed for the first time. Gamma rays interact with matter, resulting in three major processes that play an important role in radiation measurements in the intermediate energy range from 10 keV to 10 MeV: *photoelectric absorption, Compton scattering,* and *pair production.* The photoelectric effect predominates in the low energy region in high atomic number elements, while Compton scattering dominates in the medium energy range in elements with low atomic number. Also, if the gamma-ray energy exceeds twice the rest-mass energy of an electron (1.02 MeV), the process of pair production is energetically possible [ix-xiii].

The innovation of this study is to estimate the effective atomic number of polyethylene and coal using the Compton scattering.

## II. MATERIALS AND METHODS

Interaction cross section is a function of photon energy and atomic number. In other words, the interaction of photons with certain amount of energy is proportional to $Z^n$ where n is the expected value and has different values for different interactions. The value of n is defined as a value between 4 and 5 for the photoelectric effect, 1 for the Compton scattering, and 2 for pair production.

In the interaction of gamma rays with a composite, the effective atomic number is replaced by the atomic number, depending upon the radiant energy of the constituent elements. The effective atomic number for each material, is expressed by following equation [xiv]

$$Z_{eff}^{n-1} = \in \ p_i Z_i^{n-1} \qquad (1)$$

Where $Z_{eff}$ is the effective atomic number, ε is the correction factor, $P_i$ is the fractional part by weight of the whole mixture, $Z_i$ is element atomic number and n is expected value [ix]. As mentioned above, for Compton scattering the value of n is equal to 1. It is seen that Eq. (1) cannot be used to obtain the effective atomic number for Compton interaction. Consequently, the scattering cross section is used to obtain it. The scattering cross sections at an angle (θ) is given by [xiv]

$$\sigma(\theta) = \ \sigma_{kn}(\theta) S(X, Z) \qquad (2)$$

Where scattering cross section $\sigma(\theta)$ is in millibarns per atom per steradian and Klein-Nishina cross section $\sigma_{kn}(\theta)$ is in millibarn/electron/steradian. S(X, Z) is the incoherent scattering function that is an indicator of the number of electrons participating in the scattering. The momentum transferred in the scattering (X) is given by [xiv]

$$X = \frac{sin(\frac{\theta}{2})}{\lambda}(A^0) \quad (3)$$

where ν is the frequency corresponding to the incident photon. If the electrons that participate in scattering are completely released from atomic binding, S(X, Z) will be equal to the atomic number Z. Therefore, Eq. (2) can be written by the following approximation [xiv]

$$\sigma(\theta) = Z\,\sigma_{kn}(\theta) \quad (4)$$

In this equation, Z is the atomic number for a pure element and $Z_{eff}$ is the effective atomic number for the material [xiv].

In this study, following Klein-Nishina equation was used to calculate the scattering cross sections for 662 keV gamma rays emitted from the $^{137}$Cs source at angles ranging from 50° to 100 ° in steps of 10°.

$$\frac{d\sigma_{kn}(\theta)}{d\Omega} = \frac{r_0^2}{2}(\frac{E'}{E})^2\left(\frac{E'}{E} + \frac{E}{E'} - sin^2(\theta)\right) \quad (5)$$

$$\sigma_{kn}(\theta) = \frac{3}{4}\sigma_T\left[\frac{1+\alpha}{\alpha^2}\left(\frac{2(1+\alpha)}{(1+2\alpha)}\frac{ln(1+2\alpha)}{\alpha}\right) + \frac{ln(1+2\alpha)}{2\alpha}\frac{1+3\alpha}{(1+2\alpha)^2}\right] \quad [xvi] \quad (6)$$

$$\sigma_T(\theta) = \frac{r_0^2}{2}\left(1+\cos^2(\theta)\right) \quad (7)$$

$$\alpha = \frac{E}{mc^2} \quad (8)$$

In these expressions, $\sigma_T$ is the Thomson cross section, $\sigma_{kn}$ is the Klein-Nishina cross section, θ is the scattering angle, $r_0$ is the classical radius, E is the photon energy, E is the scattered photon energy and m is the electron rest mass [xvi]. The results are listed in the Table I.

TABLE I
THE RESULTS OF KLEIN-NISHINA SCATTERING CROSS SECTIONS, THE TOTAL SCATTERING CROSS SECTIONS AND THE EFFECTIVE ATOMIC NUMBER FOR DIFFERENT ANGLES

| Angle(degree) | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|
| Klein-Nishina scattering cross section | 21.61 | 19.11 | 17.08 | 15.75 | 15.28 | 15.74 |
| Total scattering cross sections for polyethylene | 52.95 | 42.00 | 42.77 | 34.16 | 33.64 | 31.47 |
| Effective atomic number for polyethylene | 2.45 | 2.20 | 2.50 | 2.17 | 2.20 | 2.00 |
| average atomic number | 2.25 | | | | | |
| Total scattering cross sections for coal | - | - | 49.68 | 19.10 | 11.87 | 73.52 |
| Effective atomic number for coal | - | - | 2.91 | 1.21 | 0.78 | 4.67 |
| average atomic number | 2.39 | | | | | |

*A. Experimental details and method of measurements*

Fig.1 shows the experimental setup of the system used to determine the cross sections. In experimental investigations, the intensity of photons scattered from polyethylene and coal are measured. Therefore, a cylinder of polyethylene with a radius of 2 cm and a height of 10 cm, and an irregularly shaped lump of coal with an average diameter of 2 cm and a height of 10 cm, were placed in the center of Compton scattering palate at a distance of 11 cm from the surface of 2in×2in NaI(Tl) detector. From $^{137}$Cs source with an activity of 100 Micro Curie (μCi), 662 keV gamma rays were emitted to the samples at angles of $50^0$ to $100^0$ in steps of $10^0$. As shown in Fig.1, a lead shield with dimensions of 10 × 10 × 5 $cm^3$ was placed between the detector and the source to avoid any energy loss as the particles travel from the source to the detector and to

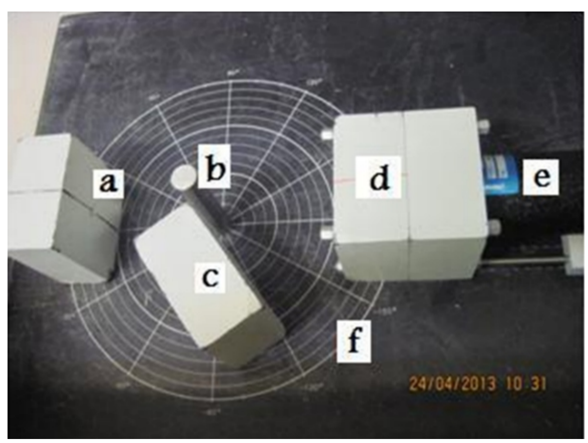


Fig.1. Photograph of experimental set up: a) $^{137}$Cs source, b)sample, c)lead shield, d)lead container, e) NaI (Tl) detector, f) Compton scattering plate.

restrict any background radiation scattered into the detector. The spectrum that recorded by MCA and Cassy lab software, was analyzed. This spectrum showed the change of counts versus channel number. It must be mentioned that the channel number bears a linear relation with photon energy. The number of photons depositing their energy in the detector can be determined by calculating the area under the spectrum. According to the following equation, these photon numbers are related to the total scattering cross section.

$$\sigma(\theta)= \int \frac{d\sigma}{d\Omega} d\Omega - \int \frac{d\sigma_0}{d\Omega}\, d\Omega \qquad (9)$$

Where $N = \frac{d\sigma}{d\Omega}$ and $N_0 = \frac{d\sigma_0}{d\Omega}$ are the scattering probability with and without the source respectively. The counting time was 20 min in all the spectrums. Therefore, Eq. 9 is converted to Eq. 10

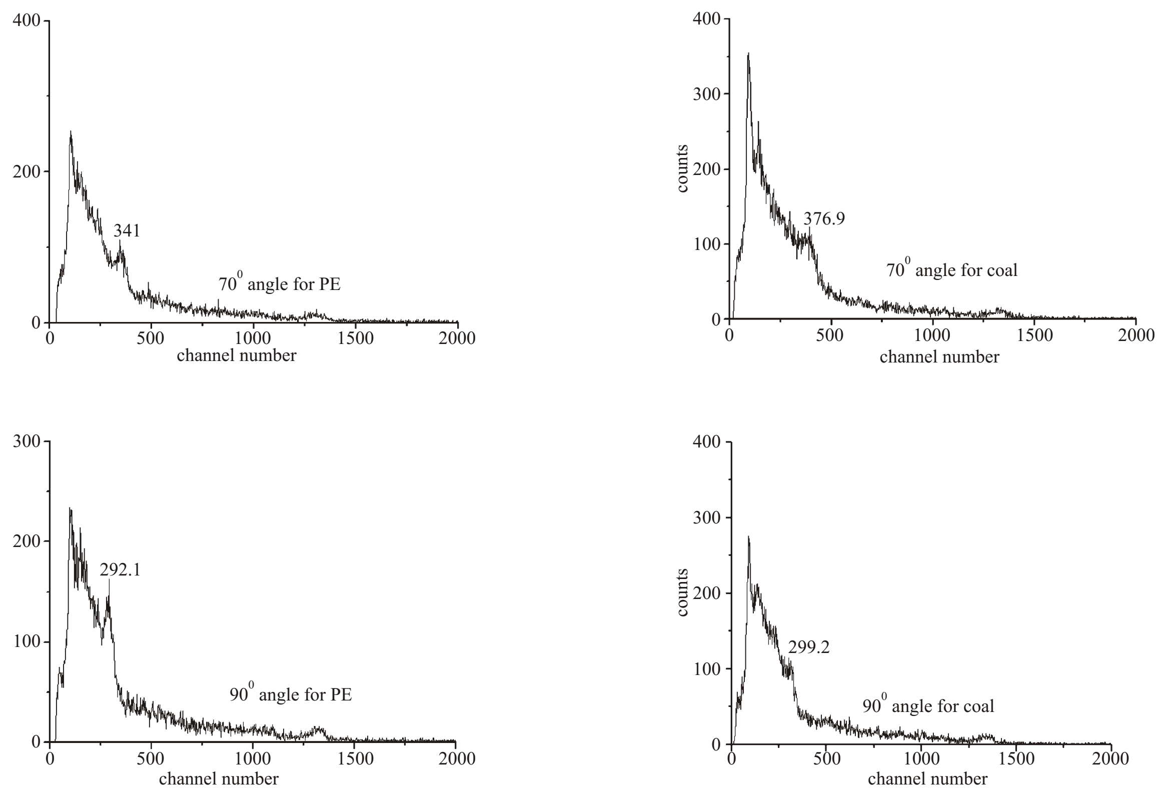
$$\sigma(\theta)=\frac{N-N_0}{1200}\,\Delta\Omega \qquad (10)$$

$\Delta\Omega$ is the ratio of the detector surface (3.14 $cm^2$) to the squared distance from the detector to the sample (121$cm^2$). For example, Fig.2 shows the spectrum from polyethylene and coal samples at angles of 70, 90, and 100 degrees.

## III. Results and Discussion

Fig. 2 shows that full-energy peaks for polyethylene are in 341, 292.1, and 245 channels at $70^0$,$80^0$, and $100^0$ angles respectively. For coal, these channels are 376.9, 299.2, and 252.1 at the same angles. It is indicated that the Compton scattering depends on the external shell electrons. Compton attenuation coefficient varies via photon energy. In this research, the photon energy is fixed at 662 KeV that emitted from $^{137}Cs$. The difference of channel numbers for polyethylene and coal at same angles shows the atomic number effects on Compton scattering.

By comparing these numbers, it is clear that full-energy peaks are obviously observed at small angles. This implies that photon intensities decrease with the increasing scattering angle of radiation due to Compton scattering. By changing the intensity, the center of the peak moves slightly. This shift shows a change in photon energy that is known as the Compton defect. Because of the decrease in scattered photon energy at large angles, the channel numbers are slightly shifted to the left in Fig. 2.



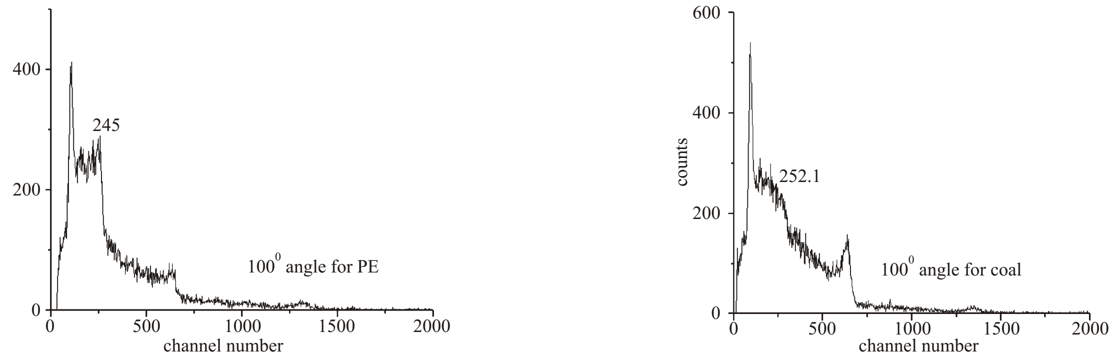



Fig.2. Spectrum of variation of counts versus channel number (energy) for polyethylene (PE) and coal at $70^0$, $90^0$ and $100^0$ angles.

The present work focuses on the calculation $Z_{eff}$ of polyethylene and coal by a direct experimental method. The results of Klein-Nishina scattering cross sections, the total scattering cross sections, and the effective atomic numbers for different angles obtained by using equations (4-10) are given in the Table I. It is calculated that the effective atomic numbers (the average atomic number) for polyethylene and coal are 2.25 and 2.39, respectively. In the present study, due to the irregular shape of coal, the obtained results are not satisfactory at angles of $50^0$ and $60^0$. We can notice that Klein-Nishina scattering calculations are in acceptable agreement. Since type and amount of elements in the samples of polyethylene and coal vary, as the amount of impurities in the material differ, no accepted values for their atomic numbers have been reported in other research studies. Thus, one limitation of this study is that the obtained results have not been compared with findings from other research.

## IV. CONCLUSION

The Compton scattering of gamma-ray photons is governed by the atomic number of material. Therefore, it is indicated that the Compton scattering can be used to estimate the effective atomic numbers ($Z_{eff}$) as useful parameter in studying of attenuation coefficient, photon interactions with matter, and radiation shielding.

This study investigated the Compton scattering of photons from stationary electrons in samples of polyethylene and coal. The effective atomic numbers of the materials were estimated by calculating the area under the spectrum of full-energy peaks. It is calculated that the effective atomic numbers of polyethylene and coal are 2.25 and 2.39, respectively.

It is recommended that in the future studies, effective atomic number of other composite be estimated by Compton scattering. In addition, it is proposed that the present study be carried out for other sources of photons.

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**Authorship and Contribution Declaration**

| | Author-s Full Name | Contribution to Paper | |
|---|---|---|---|
| 1 | Dr. D. Rezaei Och belagh (Main/Principal Author) | Proposed topic, basic study Design, methodology and manuscript writing | |
| 2 | Miss Atefah Vasseh Mosalla (2nd Author) | Data Collection, statistical analysis and interpretation of results etc. | |
| 3 | Miss Akaram Yahyabadi (3rd Author) | Literature review & Referencing and quality insurer | |